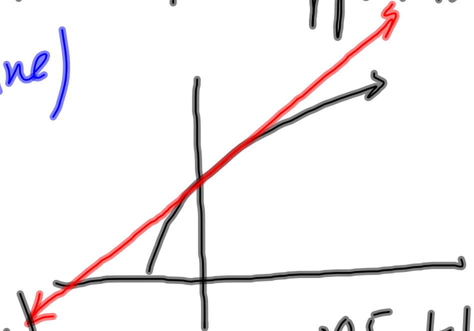
4.5 linearization - tangent line

Find the linearization to $y = \sqrt{1+x}$ at $x=0$

$y = \frac{1}{2}x + 1$

use linearizations to approximate function
(line) (curve)



approx $\sqrt{1.01}$

$x = .01$

$\sqrt{1.01} \approx \frac{1}{2}(.01) + 1 = .005 + 1 = 1.005$

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$\sqrt{5}$? $y = \sqrt{1+x}$

$x=4$

we need a value of x that is close to $x=4$ & easy to evaluate

$x=3$ $\sqrt{4} = 2$

do tan line at $x=3$ instead of $x=0$

$y' = \left.\frac{1}{2\sqrt{1+x}}\right|_{x=3} = \frac{1}{4}$ tan line $y = \frac{1}{4}(x-3)+2$

$\sqrt{5} \approx \frac{1}{4}(4-3)+2 = 2.25$

Nov 2-10:08 AM

$\cos(1.75) = ?$ $y = \cos(x)$

we need an x that easy & close

$x = \frac{\pi}{2} \approx 1.6$ is close, easy? $\cos\frac{\pi}{2} = 0$.

$y' = -\sin x$ $y'(\frac{\pi}{2}) = -\sin\frac{\pi}{2} = -1$

tan line: $y = -1(x - \frac{\pi}{2}) + 0$

$\cos(1.75) \approx -1(1.75 - \frac{\pi}{2}) = -.179204$

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approx $\sqrt{123}$ $y = \sqrt{x}$ $x = 121$

tan line $y' = \frac{1}{2\sqrt{x}}$ $y'(121) = \frac{1}{22}$

↓

$y = \frac{1}{22}(x - \underset{x_0}{121}) + \underset{y_0}{11}$

$y(123) \approx \frac{1}{22}(123 - 121) + 11 = 11\frac{2}{22}$

$= 11\frac{1}{11} \approx 11.0909...$

easy
$x = 121$
$y = 11$

hard
$x = 123$
$y \approx 11.0909...$

$\Delta y \approx .0909...$

change in y when going from easy to hard

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sometimes approx y, sometimes approx Δy

$$\boxed{y(x) \approx y'(x_0)(x-x_0) + y(x_0)}$$

x_0 easy, x hard

$$\Delta y \approx y(x) - y(x_0) = y'(x_0)(x-x_0)$$

$$\boxed{\Delta y \approx \underbrace{y'(x_0)\,\Delta x}_{dy}}$$

dy differential of y

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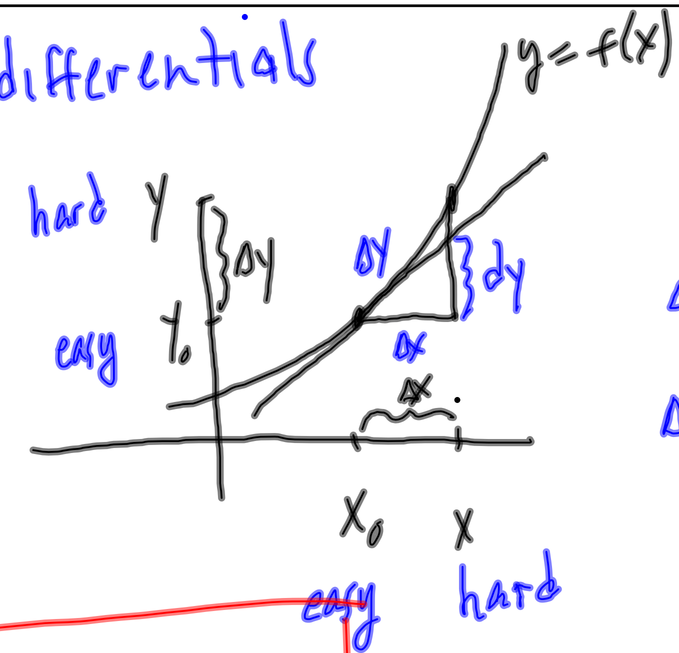
differentials



$$\Delta x = dx$$

$$\Delta y \approx dy$$

$$\frac{dy}{dx} = \frac{dy}{dx}$$

$$\boxed{dy = \frac{dy}{dx} \cdot dx}$$

$dy \div dx$

derivative

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ex 7 $y = \tan(2x)$ find $d(\tan(2x))$

$dy = ?$ $\frac{dy}{dx} dx = 2\sec^2(2x) \cdot dx$

let $x = 0$ $dx = .01$ hard $x = 0 + .01$

easy

$= .01$

$dy = (2\sec^2 0)(.01) = .02$

approx $\Delta y = \tan(2(.01)) - \tan(2 \cdot 0)$

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absolute change $\Delta y \approx f'(x_0)\,dx$

relative change $\frac{\Delta y}{y}$

percent change $\frac{\Delta y}{y} \cdot 100\%$

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Ex 10 radius $= 3959 \pm 0.1$

estimate surface area

$S = 4\pi r^2 \qquad \Delta r = dr = 0.1$

$\Delta S \approx \frac{dS}{dr} \cdot \Delta r$ error

$8\pi r \cdot (0.1) \Big|_{r=3959} = 9950$

% error $\frac{9950}{4\pi(3959)^2} \times 100\% = .0051\%$

$\frac{\Delta S}{S} \times 100\%$

$\frac{.1}{3959} \times 100\% = .0025\%$

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